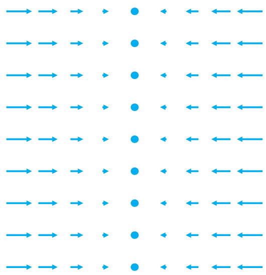
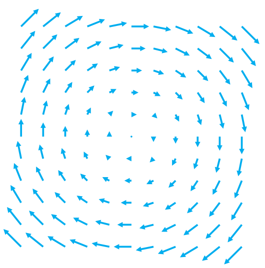
Problem 1: For parts (a)-(f), does the vector field appear to be conservative?

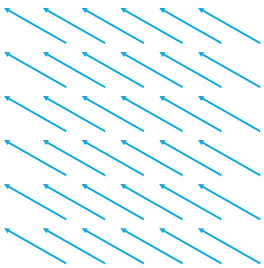
(a)



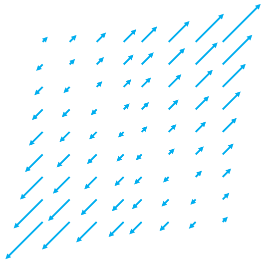
(b)



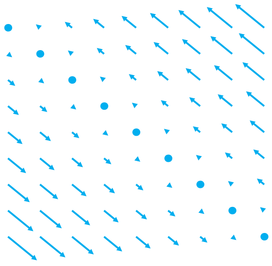
(c)



(d)



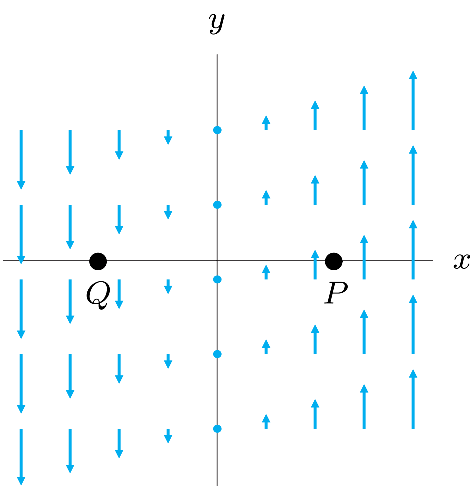
(e)



Problem 2: The figure below shows the vector field $\vec{F}(x,y) = \begin{bmatrix} 0 & x \end{bmatrix}^T$. Find paths C_1, C_2, and C_3 from P to Q such that

$$\int_{C_1} \vec{F} \cdot d\vec{r} = 0, \quad \int_{C_2} \vec{F} \cdot d\vec{r} > 0, \quad \int_{C_3} \vec{F} \cdot d\vec{r} < 0.$$

Is $\vec{F}$ a gradient field?



Problem 3: Prove that $\vec{F}(x,y) = 2xy\vec{i} + xy\vec{j}$ is not conservative.

Problem 4: Show that the field

$$\vec{F}(x,y) = \frac{-y\vec{i} + x\vec{j}}{x^2 + y^2}$$

has zero curl on its domain (the plane with the origin removed), but is *not* conservative.

Problem 5: Decide which of the following vector fields are conservative. For the ones that are conservative, find a potential function.

(a) $\vec{F}(x,y) = y\vec{i} - x\vec{j}$

(b) $\vec{F}(x,y) = 2xy\vec{i} + x^2\vec{j}$

(c) $\vec{F}(x,y) = (2xy^3 + y)\vec{i} + (3x^2y^2 + x)\vec{j}$

(d) $\vec{F}(x,y) = \dfrac{x\vec{i} + y\vec{j}}{x^2 + y^2}$