Problem 1: Sketch the following vector fields in $\mathbb{R}^2$.

(a) $\vec{F}(x,y) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) $\vec{F}(x,y) = \begin{bmatrix} y \\ 0 \end{bmatrix}$

(c) $\vec{F}(x,y) = \begin{bmatrix} x \\ y \end{bmatrix}$

(d) $\vec{F}(x,y) = \begin{bmatrix} y \\ -x \end{bmatrix}$

(e) $\vec{F}(x,y) = \dfrac{x\vec{i} + y\vec{j}}{\sqrt{x^2+y^2}}$

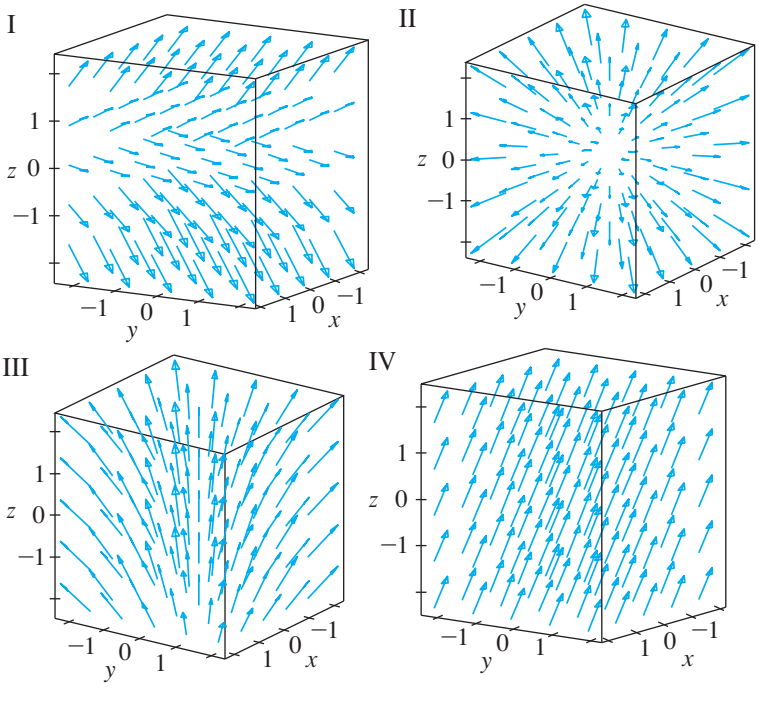
Problem 2: Match the vector fields in $\mathbb{R}^3$ to their formulas.

(a) $\vec{F}(x,y,z) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(b) $\vec{F}(x,y,z) = \begin{bmatrix} 1 \\ 2 \\ z \end{bmatrix}$

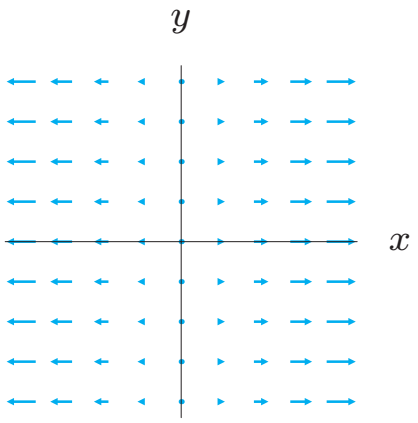
(c) $\vec{F}(x,y,z) = \begin{bmatrix} x \\ y \\ 3 \end{bmatrix}$

(d) $\vec{F}(x,y,z) = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

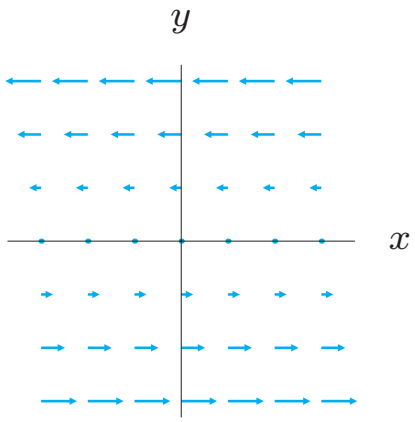


Problem 3: Find formulas for the vectors fields below.

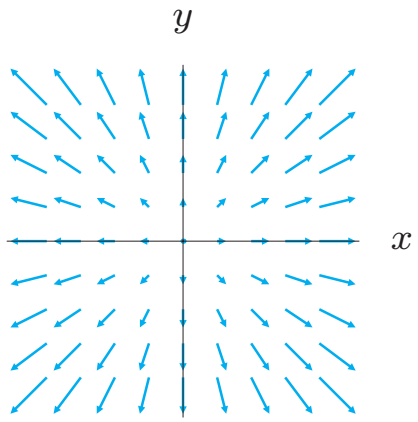
(a)



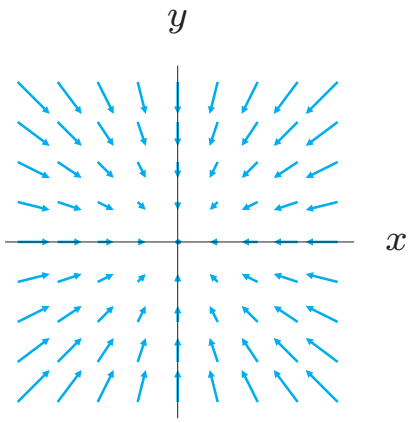
(b)



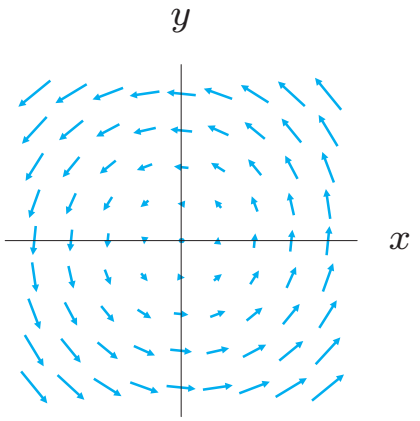
(c)



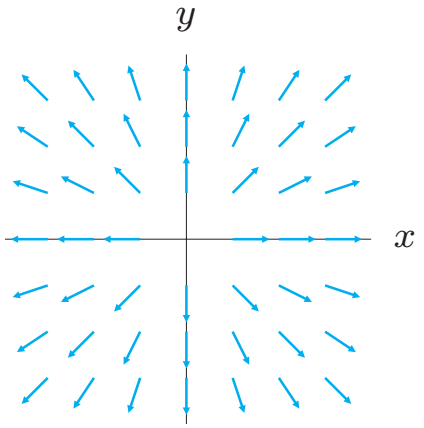
(d)



(d)



(f)



Problem 4: For each description of a vector field in parts (a)-(d), choose one or more of the vector fields (i)-(ix).

(a) Pointing radially outward, increasing in length away from the origin.

(b) Pointing in a circular direction around the origin, remaining the same length.

(c) Pointing toward the origin, increasing in length farther from the origin.

(d) Pointing clockwise around the origin.

(i) $\dfrac{1}{\sqrt{x^2+y^2}}\begin{bmatrix} x \\ y \end{bmatrix}$

(ii) $\dfrac{1}{\sqrt{x^2+y^2}}\begin{bmatrix} -y \\ x \end{bmatrix}$

(iii) $x\vec{i} + y\vec{j}$

(iv) $-x\vec{i} - y\vec{j}$

(v) $\begin{bmatrix} -y \\ x \end{bmatrix}$

(vi) $\begin{bmatrix} y \\ -x \end{bmatrix}$

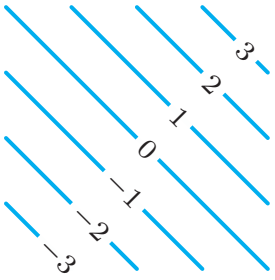
(vii) $\begin{bmatrix} y \\ x \end{bmatrix}$

(viii) $\dfrac{x\vec{i} + y\vec{j}}{(x^2+y^2)^{3/2}}$

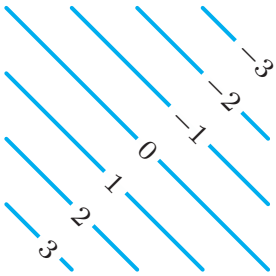
(ix) $-\dfrac{x\vec{i} + y\vec{j}}{(x^2+y^2)^{3/2}}$

Problem 5: Match the contour plots in (a)-(d) with the gradient fields in (i)-(iv).

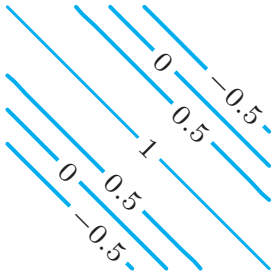
(a)



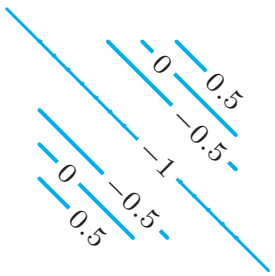
(b)



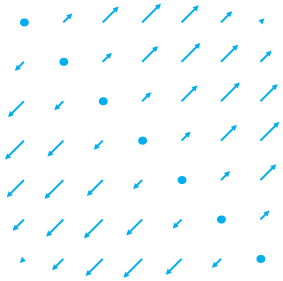
(c)



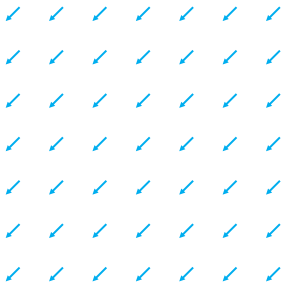
(d)



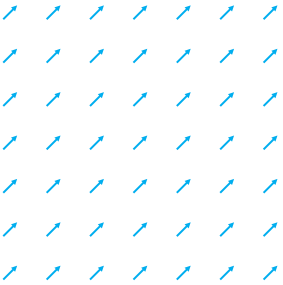
(i)



(ii)



(iii)



(iv)