Since the line integral around this closed curve is nonzero, $\vec{G}$ is path-dependent. Computations show $\text{curl}\,\vec{G} = \vec{0}$. However, the domain of $\vec{G}$ is all of 3-space minus the z-axis, and it does not satisfy the curl test domain criterion. For example, the circle, C, is lassoed around the z-axis, and cannot be pulled to a point without hitting the z-axis. Thus, the curl test does not apply.

Exercises and Problems for Section 18.4

Exercises

In Exercises 1–10, decide if the given vector field is the gradient of a function f. If so, find f. If not, explain why not.

1. $y\vec{i} - x\vec{j}$

2. $2xy\vec{i} + x^2\vec{j}$

3. $y\vec{i} + y\vec{j}$

4. $2xy\vec{i} + 2xy\vec{j}$

5. $(x^2+y^2)\vec{i} + 2xy\vec{j}$

6. $(2xy^3+y)\vec{i} + (3x^2y^2+x)\vec{j}$

7. $\dfrac{\vec{i}}{x} + \dfrac{\vec{j}}{y} + \dfrac{\vec{k}}{z}$

8. $\dfrac{\vec{i}}{x} + \dfrac{\vec{j}}{y} + \dfrac{\vec{k}}{xy}$

9. $2x\cos(x^2+z^2)\vec{i} + \sin(x^2+z^2)\vec{j} + 2z\cos(x^2+z^2)\vec{k}$

10. $\dfrac{y}{x^2+y^2}\vec{i} - \dfrac{x}{x^2+y^2}\vec{j}$

11. Use Green's Theorem to evaluate $\int_C \left(y^2\vec{i} + x\vec{j}\right)\cdot d\vec{r}$ where C is the counterclockwise path around the perimeter of the rectangle $0 \le x \le 2$, $0 \le y \le 3$.

In Exercises 12–15, use Green's Theorem to calculate the circulation of $\vec{F}$ around the curve, oriented counterclockwise.

12. $\vec{F} = y\vec{i} - x\vec{j}$ around the unit circle.

13. $\vec{F} = xy\vec{j}$ around the square $0 \le x \le 1$, $0 \le y \le 1$.

14. $\vec{F} = (2x^2+3y)\vec{i} + (2x+3y^2)\vec{j}$ around the triangle with vertices $(2,0)$, $(0,3)$, $(-2,0)$.

15. $\vec{F} = 3y\vec{i} + xy\vec{j}$ around the unit circle.

16. Calculate $\int_C ((3x+5y)\vec{i} + (2x+7y)\vec{j})\cdot d\vec{r}$ where C is the circular path with center (a,b) and radius m, oriented counterclockwise. Use Green's Theorem.

17. **(a)** Sketch $\vec{F} = y\vec{i}$ and determine the sign of the circulation of $\vec{F}$ around the unit circle centered at the origin and traversed counterclockwise.
(b) Use Green's Theorem to compute the circulation in part (a) exactly.

Problems

18. Let $\vec{F} = (\sin x)\vec{i} + (x+y)\vec{j}$. Find the line integral of $\vec{F}$ around the perimeter of the rectangle with corners $(3,0)$, $(3,5)$, $(-1,5)$, $(-1,0)$, traversed in that order.

19. Find $\displaystyle\int_C (\sin(x^2)\cos y)\vec{i} + (\sin(y^2)+e^x)\vec{j}\cdot d\vec{r}$ where C is the square of side 1 in the first quadrant of the xy-plane, with one vertex at the origin and sides along the axes, and oriented counterclockwise when viewed from above.

20. Find the line integral of $\vec{F} = (x-y)\vec{i} + x\vec{j}$ around the closed curve in Figure 18.50. (The arc is part of a circle.)

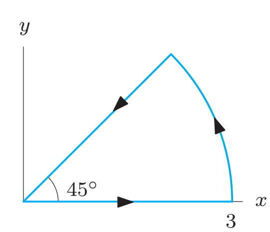

Figure 18.50

21. Find the line integral of $\vec{F} = (x+y)\vec{i} + \sin y\vec{j}$ around the closed curve in Figure 18.50. (The arc is part of a circle.)

22. Let $\vec{F} = 2xe^y\vec{i} + x^2e^y\vec{j}$ and $\vec{G} = (x-y)\vec{i} + (x+y)\vec{j}$. Let C be the line from $(0,0)$ to $(2,4)$. Find exactly:

(a) $\int_C \vec{F}\cdot d\vec{r}$ **(b)** $\int_C \vec{G}\cdot d\vec{r}$

23. Let $\vec{F} = y\vec{i} + x\vec{j}$ and $\vec{G} = 3y\vec{i} - 3x\vec{j}$. In Figure 18.51, the curve C_2 is the semicircle centered at the origin from $(-1,1)$ to $(1,-1)$ and C_1 is the line segment from $(-1,1)$ to $(1,-1)$, and $C = C_2 - C_1$. Find the following line integrals:

(a) $\displaystyle\int_{C_1}\vec{F}\cdot d\vec{r}$ **(b)** $\displaystyle\int_{C}\vec{F}\cdot d\vec{r}$

(c) $\displaystyle\int_{C_2}\vec{F}\cdot d\vec{r}$ **(d)** $\displaystyle\int_{C_2}\vec{G}\cdot d\vec{r}$

(e) $\displaystyle\int_{C}\vec{G}\cdot d\vec{r}$ **(f)** $\displaystyle\int_{C_1}\vec{G}\cdot d\vec{r}$

(g) $\displaystyle\int_{C}(\vec{F}+\vec{G})\cdot d\vec{r}$