If C_1 and C_2 are oriented curves with C_1 ending where C_2 begins, we construct a new oriented curve, called $C_1 + C_2$, by joining them together. (See Figure 18.12.) Property 4 is the analogue for line integrals of the property for definite integrals which says that

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

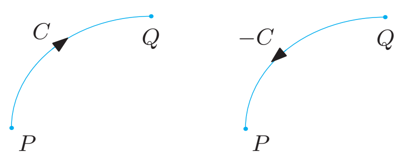


Figure 18.11: A curve, C, and its opposite, $-C$

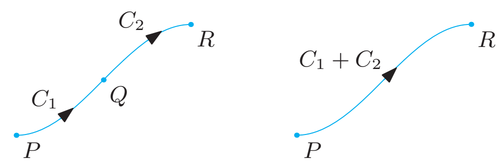


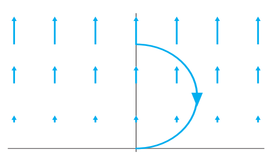
Figure 18.12: Joining two curves, C_1, and C_2, to make a new one, $C_1 + C_2$

Exercises and Problems for Section 18.1

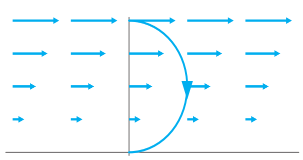
Exercises

In Exercises 1–6, say whether you expect the line integral of the pictured vector field over the given curve to be positive, negative, or zero.

1.



2.



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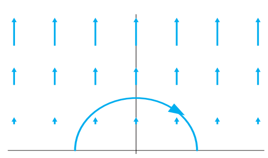

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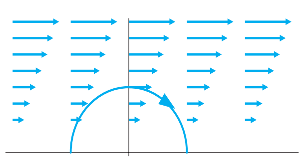

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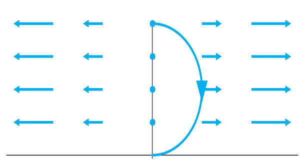
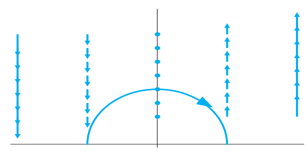


6.



In Exercises 7–15, calculate the line integral of the vector field along the line between the given points.

7. $\vec{F} = x\vec{j}$, from $(1,0)$ to $(3,0)$

8. $\vec{F} = x\vec{j}$, from $(2,0)$ to $(2,5)$

9. $\vec{F} = 6\vec{i} - 7\vec{j}$, from $(0,0)$ to $(7,6)$

10. $\vec{F} = 6\vec{i} + y^2\vec{j}$, from $(3,0)$ to $(7,0)$

11. $\vec{F} = 3\vec{i} + 4\vec{j}$, from $(0,6)$ to $(0,13)$

12. $\vec{F} = x\vec{i}$, from $(2,0)$ to $(6,0)$

13. $\vec{F} = x\vec{i} + y\vec{j}$, from $(2,0)$ to $(6,0)$

14. $\vec{F} = \vec{r} = x\vec{i} + y\vec{j}$, from $(2,2)$ to $(6,6)$

15. $\vec{F} = x\vec{i} + 6\vec{j} - \vec{k}$, from $(0,-2,0)$ to $(0,-10,0)$

In Exercises 16–18, find $\int_C \vec{F} \cdot d\vec{r}$ for the given $\vec{F}$ and C.

16. $\vec{F} = 5\vec{i} + 7\vec{j}$, and C is the x-axis from $(-1,0)$ to $(-9,0)$.

17. $\vec{F} = x^2\vec{i} + y^2\vec{j}$, and C is the x-axis from $(2,0)$ to $(3,0)$.

18. $\vec{F} = 6x\vec{i} + (x+y^2)\vec{j}$; C is the y-axis from $(0,3)$ to $(0,5)$.

In Exercises 19–22, calculate the line integral.

19. $\int_C (2\vec{j} + 3\vec{k}) \cdot d\vec{r}$ where C is the y-axis from the origin to the point $(0, 10, 0)$.

20. $\int_C (2x\vec{i} + 3y\vec{j}) \cdot d\vec{r}$, where C is the line from $(1, 0, 0)$ to $(1, 0, 5)$.

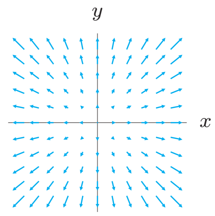
21. $\int_C ((2y + 7)\vec{i} + 3x\vec{j}) \cdot d\vec{r}$, where C is the line from $(1, 0, 0)$ to $(5, 0, 0)$.

22. $\int_C (x\vec{i} + y\vec{j} + z\vec{k}) \cdot d\vec{r}$ where C is the unit circle in the xy-plane, oriented counterclockwise.

Problems

In Problems 23–26, let C_1 be the line from $(0, 0)$ to $(0, 1)$; let C_2 be the line from $(1, 0)$ to $(0, 1)$; let C_3 be the semicircle in the upper half plane from $(-1, 0)$ to $(1, 0)$. Do the line integrals of the vector field along each of the paths C_1, C_2, and C_3 appear to be positive, negative, or zero?

23.



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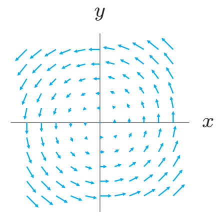

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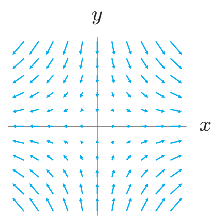

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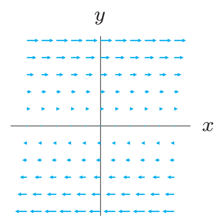

27. Consider the vector field $\vec{F}$ shown in Figure 18.13, together with the paths C_1, C_2, and C_3. Arrange the line integrals $\int_{C_1} \vec{F} \cdot d\vec{r}$, $\int_{C_2} \vec{F} \cdot d\vec{r}$ and $\int_{C_3} \vec{F} \cdot d\vec{r}$ in ascending order.

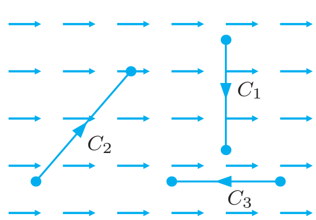

Figure 18.13

28. Compute $\int_C \vec{F} \cdot d\vec{r}$, where C is the oriented curve in Figure 18.14 and $\vec{F}$ is a vector field constant on each of the three straight segments of C:

$$\vec{F} = \begin{cases} \vec{i} & \text{on } PQ \\ 2\vec{i} - \vec{j} & \text{on } QR \\ 3\vec{i} + \vec{j} & \text{on } RS. \end{cases}$$

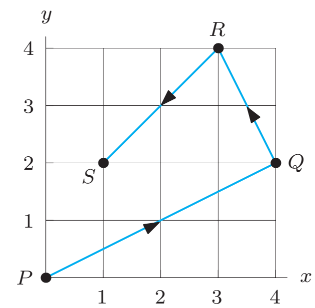

Figure 18.14

29. An object moves along the curve C in Figure 18.15 while being acted on by the force field $\vec{F}(x, y) = y\vec{i} + x^2\vec{j}$.

(a) Evaluate $\vec{F}$ at the points $(0, -1)$, $(1, -1)$, $(2, -1)$, $(3, -1)$, $(4, -1)$, $(4, 0)$, $(4, 1)$, $(4, 2)$, $(4, 3)$.

(b) Make a sketch showing the force field along C.

(c) Find the work done by $\vec{F}$ on the object.

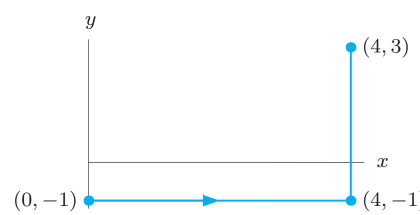

Figure 18.15

30. Let $\vec{F}$ be the constant force field $\vec{j}$ in Figure 18.16. On which of the paths C_1, C_2, C_3 is zero work done by $\vec{F}$? Explain.

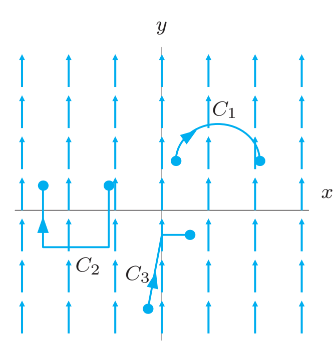

Figure 18.16