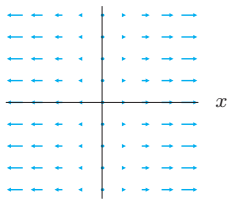
Exercises and Problems for Section 17.3

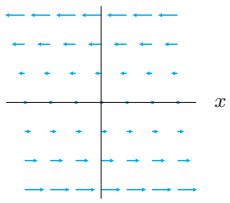
Exercises

For Exercises 1–6, find formulas for the vector fields. (There are many possible answers.)

1.



2.



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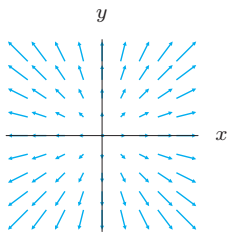

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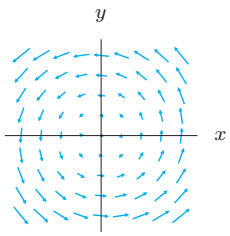

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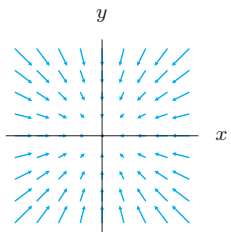

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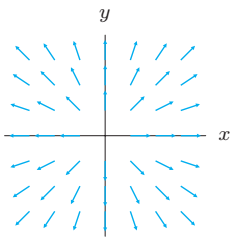


In Exercises 7–10, assume $x, y > 0$ and decide if

(a) The vector field is parallel to the x-axis, parallel to the y-axis, or neither.

(b) As x increases, the length increases, decreases, or neither.

(c) As y increases, the length increases, decreases, or neither.

Assume $x, y > 0$.

7. $\vec{F} = x\vec{j}$

8. $\vec{F} = y\vec{i} + \vec{j}$

9. $\vec{F} = (x + e^{1-y})\vec{i}$

10. $\text{grad}(x^4 + e^{3y})$

Sketch the vector fields in Exercises 11–19 in the xy-plane.

11. $\vec{F}(x, y) = 2\vec{i} + 3\vec{j}$

12. $\vec{F}(x, y) = y\vec{i}$

13. $\vec{F}(x, y) = -y\vec{j}$

14. $\vec{F}(\vec{r}) = 2\vec{r}$

15. $\vec{F}(\vec{r}) = \vec{r}/\|\vec{r}\|$

16. $\vec{F}(\vec{r}) = -\vec{r}/\|\vec{r}\|^3$

17. $\vec{F} = y\vec{i} - x\vec{j}$

18. $\vec{F}(x, y) = 2x\vec{i} + x\vec{j}$

19. $\vec{F}(x, y) = (x + y)\vec{i} + (x - y)\vec{j}$

20. For each description of a vector field in (a)-(d), choose one or more of the vector fields I-IX.

(a) Pointing radially outward, increasing in length away from the origin.

(b) Pointing in a circular direction around the origin, remaining the same length.

(c) Pointing towards the origin, increasing in length farther from the origin.

(d) Pointing clockwise around the origin.

I. $\dfrac{x\vec{i} + y\vec{j}}{\sqrt{x^2 + y^2}}$ II. $\dfrac{-y\vec{i} + x\vec{j}}{\sqrt{x^2 + y^2}}$ III. $\vec{r}$

IV. $-\vec{r}$ V. $-y\vec{i} + x\vec{j}$ VI. $y\vec{i} - x\vec{j}$

VII. $y\vec{i} + x\vec{j}$ VIII. $\dfrac{\vec{r}}{\|\vec{r}\|^3}$ IX. $-\dfrac{\vec{r}}{\|\vec{r}\|^3}$

21. Each vector field in Figures (I)–(IV) represents the force on a particle at different points in space as a result of another particle at the origin. Match up the vector fields with the descriptions below.

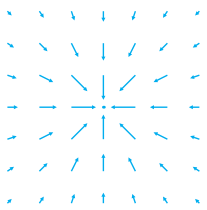
(a) A repulsive force whose magnitude decreases as distance increases, such as between electric charges of the same sign.

(b) A repulsive force whose magnitude increases as distance increases.

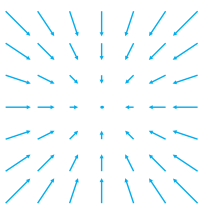
(c) An attractive force whose magnitude decreases as distance increases, such as gravity.

(d) An attractive force whose magnitude increases as distance increases.

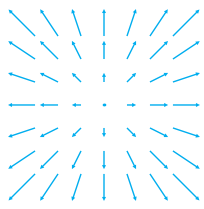
(I)



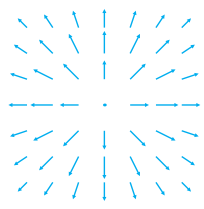
(II)



(III)



(IV)



Problems

In Problems 22–26, give an example of a vector field $\vec{F}(x, y)$ in 2-space with the stated properties.

22. $\vec{F}$ is constant

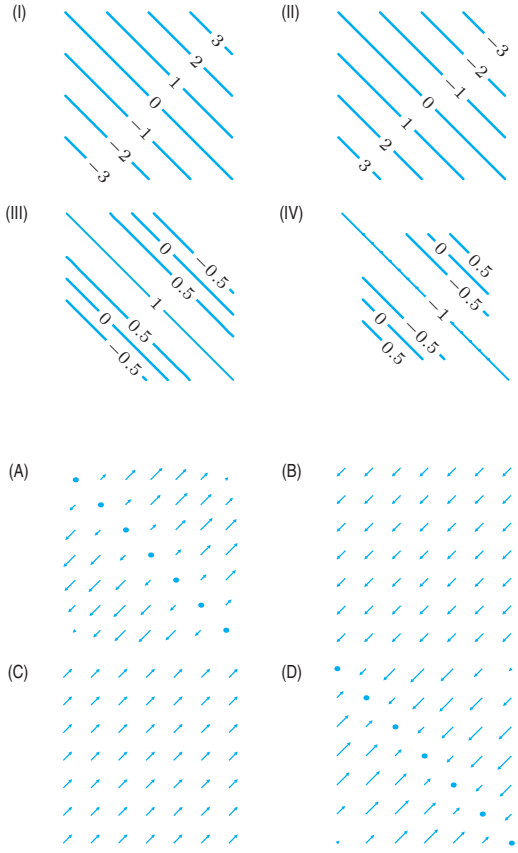
23. $\vec{F}$ has a constant direction but $\|\vec{F}\|$ is not constant

24. $\|\vec{F}\|$ is constant but $\vec{F}$ is not constant

25. Neither $\|\vec{F}\|$ nor the direction of $\vec{F}$ is constant

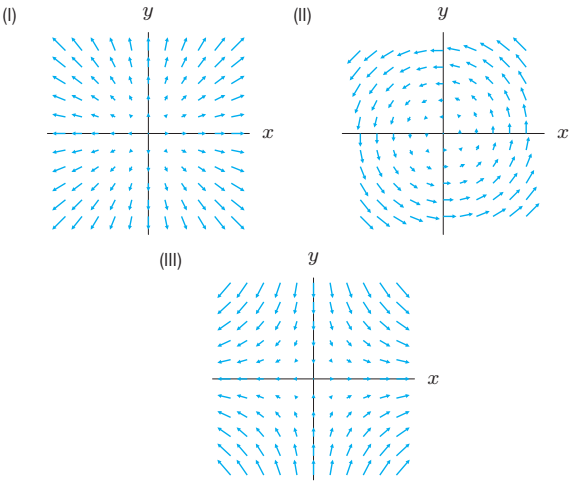
26. $\vec{F}$ is perpendicular to $\vec{G} = (x + y)\vec{i} + (1 + y^2)\vec{j}$ at every point

27. Match the level curves in (I)–(IV) with the gradient fields in (A)–(D). All figures use the same square window.



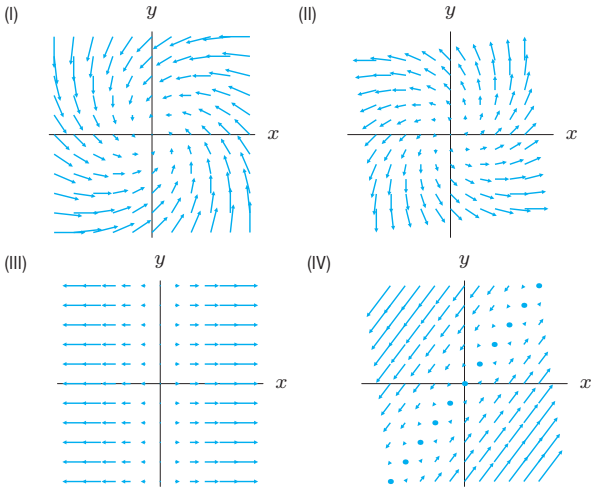
Problems 28–29 concern the vector fields $\vec{F} = x\vec{i} + y\vec{j}$, $\vec{G} = -y\vec{i} + x\vec{j}$, and $\vec{H} = x\vec{i} - y\vec{j}$.

28. Match $\vec{F}$, $\vec{G}$, $\vec{H}$ with their sketches in (I)–(III).



29. Match the vector fields with their sketches, (I)–(IV).

(a) $\vec{F} + \vec{G}$ **(b)** $\vec{F} + \vec{H}$ **(c)** $\vec{G} + \vec{H}$ **(d)** $-\vec{F} + \vec{G}$



In Problems 30–32, write formulas for vector fields with the given properties.

30. All vectors are parallel to the x-axis; all vectors on a vertical line have the same magnitude.

31. All vectors point toward the origin and have constant length.

32. All vectors are of unit length and perpendicular to the position vector at that point.

33. **(a)** Let $\vec{F} = x\vec{i} + (x + y)\vec{j} + (x - y + z)\vec{k}$. Find a point at which $\vec{F}$ is parallel to l, the line $x = 5 + t$, $y = 6 - 2t$, $z = 7 - 3t$.