## Exercises and Problems for Section 17.2

### Exercises

In Exercises 1–6, find the velocity and acceleration vectors.

1. $x = 2 + 3t, y = 4 + t, z = 1 - t$
2. $x = 2 + 3t^2, y = 4 + t^2, z = 1 - t^2$
3. $x = t, y = t^2, z = t^3$
4. $x = t, y = t^3 - t$
5. $x = 3\cos t, y = 4\sin t$
6. $x = 3\cos(t^2), y = 3\sin(t^2), z = t^2$

In Exercises 7–12, find the velocity $\vec{v}(t)$ and speed $\|\vec{v}(t)\|$. Find any times at which the particle stops.

7. $x = t, y = t^2, z = t^3$
8. $x = \cos 3t, y = \sin 5t$
9. $x = 3t^2, y = t^3 + 1$
10. $x = (t-1)^2, y = 2, z = 2t^3 - 3t^2$
11. $x = 3\sin(t^2) - 1, y = 3\cos(t^2)$
12. $x = 3\sin^2 t, y = \cos t - 1, \quad z = t^2$

In Exercises 13–16, find the length of the curve.

13. $x = 3 + 5t, y = 1 + 4t, z = 3 - t$ for $1 \le t \le 2$. Check by calculating the length by another method.
14. $x = \cos 3t, y = \sin 5t$ for $0 \le t \le 2\pi$.
15. $x = \cos(e^t), y = \sin(e^t)$ for $0 \le t \le 1$. Check by calculating the length by another method.
16. $\vec{r}(t) = 2t\vec{i} + \ln t\vec{j} + t^2\vec{k}$ for $1 \le t \le 2$.

In Exercises 17–18, find the velocity and acceleration vectors of the uniform circular motion and check that they are perpendicular. Check that the speed and magnitude of the acceleration are constant.

17. $x = 3\cos(2\pi t), y = 3\sin(2\pi t), z = 0$
18. $x = 2\pi, y = 2\sin(3t), z = 2\cos(3t)$

In Exercises 19–20, find the velocity and acceleration vectors of the straight-line motion. Check that the acceleration vector points in the same direction as the velocity vector if the speed is increasing and in the opposite direction if the speed is decreasing.

19. $x = 2 + t^2, y = 3 - 2t^2, z = 5 - t^2$
20. $x = -2t^3 - 3t + 1, y = 4t^3 + 6t - 5, z = 6t^3 + 9t - 2$
21. Find parametric equations for the tangent line at $t = 2$ for Exercise 10.

### Problems

22. A particle passes through the point $P = (5, 4, -2)$ at time $t = 4$, moving with constant velocity $\vec{v} = 2\vec{i} - 3\vec{j} + \vec{k}$. Find a parametric equation for its motion.

In Problems 23–24, find all values of $t$ for which the particle is moving parallel to the $x$-axis and to the $y$-axis. Determine the end behavior and graph the particle's path.

23. $x = t^2 - 6t, \quad y = t^3 - 3t$
24. $x = t^3 - 12t, \quad y = t^2 + 10t$
25. The table gives $x$ and $y$ coordinates of a particle in the plane at time $t$. Assuming that the particle moves smoothly and that the points given show all the major features of the motion, estimate the following quantities:
    - **(a)** The velocity vector and speed at time $t = 2$.
    - **(b)** Any times when the particle is moving parallel to the $y$-axis.
    - **(c)** Any times when the particle has come to a stop.

| $t$ | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|---|---|---|
| $x$ | 1 | 4 | 6 | 7 | 6 | 3 | 2 | 3 | 5 |
| $y$ | 3 | 2 | 3 | 5 | 8 | 10 | 11 | 10 | 9 |

26. A particle starts at the point $P = (3, 2, -5)$ and moves along a straight line toward $Q = (5, 7, -2)$ at a speed of 5 cm/sec. Let $x, y, z$ be measured in centimeters.
    - **(a)** Find the particle's velocity vector.
    - **(b)** Find parametric equations for the particle's motion.
27. A particle moves at a constant speed along a line from the point $P = (2, -1, 5)$ at time $t = 0$ to the point $Q = (5, 3, -1)$. Find parametric equations for the particle's motion if:
    - **(a)** The particle takes 5 seconds to move from $P$ to $Q$.
    - **(b)** The speed of the particle is 5 units per second.
28. A particle travels along the line $x = 1 + t, \ y = 5 + 2t, \ z = -7 + t$, where $t$ is in seconds and $x, y, z$ are in meters.
    - **(a)** When and where does the particle hit the plane $x + y + z = 1$?
    - **(b)** How fast is the particle going when it hits the plane? Give units.
29. A stone is thrown from a rooftop at time $t = 0$ seconds. Its position at time $t$ is given by

$$\vec{r}(t) = 10t\vec{i} - 5t\vec{j} + (6.4 - 4.9t^2)\vec{k}.$$

The origin is at the base of the building, which is standing on flat ground. Distance is measured in meters. The vector $\vec{i}$ points east, $\vec{j}$ points north, and $\vec{k}$ points up.

**(a)** How high is the rooftop above the ground?
**(b)** At what time does the stone hit the ground?
**(c)** How fast is the stone moving when it hits the ground?
**(d)** Where does the stone hit the ground?
**(e)** What is the stone's acceleration when it hits the ground?

**30.** A child wanders slowly down a circular staircase from the top of a tower. With $x, y, z$ in feet and the origin at the base of the tower, her position $t$ minutes from the start is given by

$$x = 10\cos t, \quad y = 10\sin t, \quad z = 90 - 5t.$$

**(a)** How tall is the tower?
**(b)** When does the child reach the bottom?
**(c)** What is her speed at time $t$?
**(d)** What is her acceleration at time $t$?

**31.** The origin is on flat ground and the $z$-axis points upward. For time $0 \le t \le 10$ in seconds and distance in centimeters, a particle moves along a path given by

$$\vec{r} = 2t\vec{i} + 3t\vec{j} + (100 - (t-5)^2)\vec{k}\,.$$

**(a)** When is the particle at the highest point? What is that point?
**(b)** When in the interval $0 \le t \le 10$ is the particle moving fastest? What is its speed at that moment?
**(c)** When in the interval $0 \le t \le 10$ is the particle moving slowest? What is its speed at that moment?

**32.** The function $w = f(x, y, z)$ has $\text{grad}\, f(7, 2, 5) = 4\vec{i} - 3\vec{j} + \vec{k}$. A particle moves along the curve $\vec{r}\,(t)$ arriving at the point $(7, 2, 5)$ with velocity $2\vec{i} + 3\vec{j} + 6\vec{k}$ when $t = 0$. Find the rate of change of $w$ with respect to time at $t = 0$.

**33.** Suppose $x$ measures horizontal distance in meters, and $y$ measures distance above the ground in meters. At time $t = 0$ in seconds, a projectile starts from a point $h$ meters above the origin with speed $v$ meters/sec at an angle $\theta$ to the horizontal. Its path is given by

$$x = (v\cos\theta)t, \quad y = h + (v\sin\theta)t - \frac{1}{2}gt^2.$$

Using this information about a general projectile, analyze the motion of a ball which travels along the path

$$x = 20t, \quad y = 2 + 25t - 4.9t^2.$$

**(a)** When does the ball hit the ground?
**(b)** Where does the ball hit the ground?
**(c)** At what height above the ground does the ball start?
**(d)** What is the value of $g$, the acceleration due to gravity?
**(e)** What are the values of $v$ and $\theta$?

**34.** A particle is moving on a path in the $xz$-plane given by $x = 20t$, $z = 5t - 0.5t^2$, where $z$ is the height of the particle above the ground in meters, $x$ is the horizontal distance in meters, and $t$ is time in seconds.

**(a)** What is the equation of the path in terms of $x$ and $z$ only?
**(b)** When is the particle at ground level?
**(c)** What is the velocity of the particle at time $t$?
**(d)** What is the speed of the particle at time $t$?
**(e)** Is the speed ever 0?
**(f)** When is the particle at the highest point?

**35.** The base of a 20-meter tower is at the origin; the base of a 20-meter tree is at $(0, 20, 0)$. The ground is flat and the $z$-axis points upward. The following parametric equations describe the motion of six projectiles each launched at time $t = 0$ in seconds.

(I) $\vec{r}\,(t) = (20 + t^2)\vec{k}$
(II) $\vec{r}\,(t) = 2t^2\vec{j} + 2t^2\vec{k}$
(III) $\vec{r}\,(t) = 20\vec{i} + 20\vec{j} + (20 - t^2)\vec{k}$
(IV) $\vec{r}\,(t) = 2t\vec{j} + (20 - t^2)\vec{k}$
(V) $\vec{r}\,(t) = (20 - 2t)\vec{i} + 2t\vec{j} + (20 - t)\vec{k}$
(VI) $\vec{r}\,(t) = t\vec{i} + t\vec{j} + t\vec{k}$

**(a)** Which projectile is launched from the top of the tower and goes downward? When and where does it hit the ground?
**(b)** Which projectile hits the top of the tree? When? From where is it launched?
**(c)** Which projectile is not launched from somewhere on the tower and hits the tree? Where and when does it hit the tree?

**36.** A particle moves on a circle of radius 5 cm, centered at the origin, in the $xy$-plane ($x$ and $y$ measured in centimeters). It starts at the point $(0, 5)$ and moves counterclockwise, going once around the circle in 8 seconds.

**(a)** Write a parameterization for the particle's motion.
**(b)** What is the particle's speed? Give units.

**37.** Determine the position vector $\vec{r}\,(t)$ for a rocket which is launched from the origin at time $t = 0$ seconds, reaches its highest point of $(x, y, z) = (1000, 3000, 10{,}000)$, where $x, y, z$ are in meters, and after the launch is subject only to the acceleration due to gravity, 9.8 m/sec$^2$.

**38.** Emily is standing on the outer edge of a merry-go-round, 10 meters from the center. The merry-go-round completes one full revolution every 20 seconds. As Emily passes over a point $P$ on the ground, she drops a ball from 3 meters above the ground.

**(a)** How fast is Emily going?
**(b)** How far from $P$ does the ball hit the ground? (The acceleration due to gravity is 9.8 m/sec$^2$.)
**(c)** How far from Emily does the ball hit the ground?