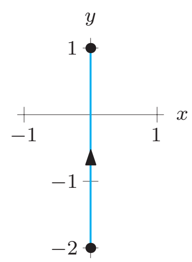
Exercises and Problems for Section 17.1

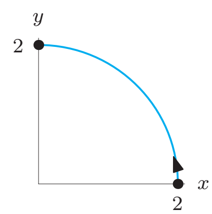
Exercises

In Exercises 1–6, find a parameterization for the curve shown.

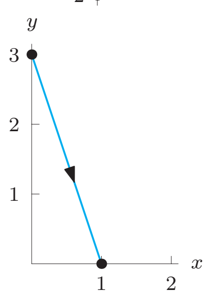
1.



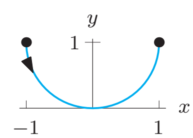
2.



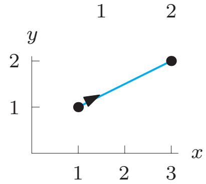
3.



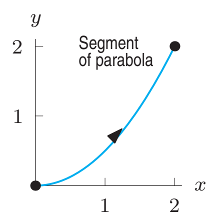
4.



5.



6.



In Exercises 7–17, find parametric equations for the line.

7. The line in the direction of the vector $\vec{i} - \vec{k}$ and through the point $(0, 1, 0)$.

8. The line in the direction of the vector $\vec{i} + 2\vec{j} - \vec{k}$ and through the point $(3, 0, -4)$.

9. The line parallel to the z-axis passing through the point $(1, 0, 0)$.

10. The line in the direction of the vector $5\vec{j} + 2\vec{k}$ and through the point $(5, -1, 1)$.

11. The line in the direction of the vector $3\vec{i} - 3\vec{j} + \vec{k}$ and through the point $(1, 2, 3)$.

12. The line in the direction of the vector $2\vec{i} + 2\vec{j} - 3\vec{k}$ and through the point $(-3, 4, -2)$.

13. The line through $(-3, -2, 1)$ and $(-1, -3, -1)$.

14. The line through the points $(1, 5, 2)$ and $(5, 0, -1)$.

15. The line through the points $(2, 3, -1)$ and $(5, 2, 0)$.

16. The line through $(3, -2, 2)$ and intersecting the y-axis at $y = 2$.

17. The line intersecting the x-axis at $x = 3$ and the z-axis at $z = -5$.

In Exercises 18–34, find a parameterization for the curve.

18. A line segment between $(2, 1, 3)$ and $(4, 3, 2)$.

19. A circle of radius 3 centered on the z-axis and lying in the plane $z = 5$.

20. A line perpendicular to the plane $z = 2x - 3y + 7$ and through the point $(1, 1, 6)$.

21. The circle of radius 2 in the xy-plane, centered at the origin, clockwise.

22. The circle of radius 2 parallel to the xy-plane, centered at the point $(0, 0, 1)$, and traversed counterclockwise when viewed from below.

23. The circle of radius 2 in the xz-plane, centered at the origin.

24. The circle of radius 3 parallel to the xy-plane, centered at the point $(0, 0, 2)$.

25. The circle of radius 3 in the yz-plane, centered at the point $(0, 0, 2)$.

26. The circle of radius 5 parallel to the yz-plane, centered at the point $(-1, 0, -2)$.

27. The curve $x = y^2$ in the xy-plane.

28. The curve $y = x^3$ in the xy-plane.

29. The curve $x = -3z^2$ in the xz-plane.

30. The curve in which the plane $z = 2$ cuts the surface $z = \sqrt{x^2 + y^2}$.

31. The curve $y = 4 - 5x^4$ through the point $(0, 4, 4)$, parallel to the xy-plane.

32. The ellipse of major diameter 5 parallel to the y-axis and minor diameter 2 parallel to the z-axis, centered at $(0, 1, -2)$.

33. The ellipse of major diameter 6 along the x-axis and minor diameter 4 along the y-axis, centered at the origin.

34. The ellipse of major diameter 3 parallel to the x-axis and minor diameter 2 parallel to the z-axis, centered at $(0, 1, -2)$.

In Exercises 35–42, find a parametric equation for the curve segment. (There are many possible answers.)

35. Line from $(-1, 2, -3)$ to $(2, 2, 2)$.

36. Line from $P_0 = (-1, -3)$ to $P_1 = (5, 2)$.

37. Line from $P_0 = (1, -3, 2)$ to $P_1 = (4, 1, -3)$.

38. Semicircle from $(0, 0, 5)$ to $(0, 0, -5)$ in the yz-plane with $y \geq 0$.

39. Semicircle from $(1, 0, 0)$ to $(-1, 0, 0)$ in the xy-plane with $y \geq 0$.

40. Graph of $y = \sqrt{x}$ from $(1, 1)$ to $(16, 4)$.

41. Arc of a circle of radius 5 from $P = (0, 0)$ to $Q = (10, 0)$.

42. Quarter-ellipse from $(4, 0, 3)$ to $(0, -3, 3)$ in the plane $z = 3$.

Problems

In Problems 43–47, parameterize the line through $P = (2, 5)$ and $Q = (12, 9)$ so that the points P and Q correspond to the given parameter values.

43. $t = 0$ and 1

44. $t = 0$ and 5

45. $t = 20$ and 30

46. $t = 10$ and 11

47. $t = 0$ and -1

48. At the point where $t = -1$, find an equation for the plane perpendicular to the line

$$x = 5 - 3t, \quad y = 5t - 7, \quad \frac{z}{t} = 6.$$

49. Determine whether the following line is parallel to the plane $2x - 3y + 5z = 5$:

$$x = 5 + 7t, \quad y = 4 + 3t, \quad z = -3 - 2t.$$

50. Show that the equations $x = 3 + t$, $y = 2t$, $z = 1 - t$ satisfy the equations $x + y + 3z = 6$ and $x - y - z = 2$. What does this tell you about the curve parameterized by these equations?

51. **(a)** Explain why the line of intersection of two planes must be parallel to the cross product of a normal vector to the first plane and a normal vector to the second.
(b) Find a vector parallel to the line of intersection of the two planes $x + 2y - 3z = 7$ and $3x - y + z = 0$.
(c) Find parametric equations for the line in part (b).

52. Find an equation for the plane containing the point $(2, 3, 4)$ and the line $x = 1 + 2t, y = 3 - t, z = 4 + t$.

53. **(a)** Find an equation for the line perpendicular to the plane $2x - 3y = z$ and through the point $(1, 3, 7)$.
(b) Where does the line cut the plane?
(c) What is the distance between the point $(1, 3, 7)$ and the plane?

54. Consider two points P_0 and P_1 in 3-space.

(a) Show that the line segment from P_0 to P_1 can be parametrized by

$$\vec{r}(t) = (1 - t)\overrightarrow{OP_0} + t\overrightarrow{OP_1}, \quad 0 \le t \le 1.$$

(b) What is represented by the parametric equation

$$\vec{r}(t) = t\overrightarrow{OP_0} + (1 - t)\overrightarrow{OP_1}, \quad 0 \le t \le 1?$$

55. **(a)** Find a vector parallel to the line of intersection of the planes $2x - y - 3z = 0$ and $x + y + z = 1$.
(b) Show that the point $(1, -1, 1)$ lies on both planes.
(c) Find parametric equations for the line of intersection.

56. Find the intersection of the line $x = 5 + 7t$, $y = 4 + 3t$, $z = -3 - 2t$ and the plane $2x - 3y + 5z = -7$.

In Problems 57–59 two parameterized lines are given. Are they the same line?

57. $\vec{r}_1(t) = (5 - 3t)\vec{i} + 2t\vec{j} + (7 + t)\vec{k}$
$\vec{r}_2(t) = (5 - 6t)\vec{i} + 4t\vec{j} + (7 + 3t)\vec{k}$

58. $\vec{r}_1(t) = (5 - 3t)\vec{i} + (1 + t)\vec{j} + 2t\vec{k}$
$\vec{r}_2(t) = (2 + 6t)\vec{i} + (2 - 2t)\vec{j} + (2 - 4t)\vec{k}$

59. $\vec{r}_1(t) = (5 - 3t)\vec{i} + (1 + t)\vec{j} + 2t\vec{k}$
$\vec{r}_2(t) = (2 + 6t)\vec{i} + (2 - 2t)\vec{j} + (3 - 4t)\vec{k}$

60. If it exists, find the value of c for which the lines $l(t) = (c + t, 1 + t, 5 + t)$ and $m(s) = (s, 1 - s, 3 + s)$ intersect.

61. **(a)** Where does the line $\vec{r} = 2\vec{i} + 5\vec{j} + t(3\vec{i} + \vec{j} + 2\vec{k})$ cut the plane $x + y + z = 1$?
(b) Find a vector perpendicular to the line and lying in the plane.
(c) Find an equation for the line that passes through the point of intersection of the line and plane, is perpendicular to the line, and lies in the plane.

In Problems 62–65, find parametric equations for the line.

62. The line of intersection of the planes $x - y + z = 3$ and $2x + y - z = 5$.

63. The line of intersection of the planes $x + y + z = 3$ and $x - y + 2z = 2$.

64. The line perpendicular to the surface $z = x^2 + y^2$ at the point $(1, 2, 5)$.

65. The line through the point $(-4, 2, 3)$ and parallel to a line in the yz-plane which makes a 45° angle with the positive y-axis and the positive z-axis.

66. Is the point $(-3, -4, 2)$ visible from the point $(4, 5, 0)$ if there is an opaque ball of radius 1 centered at the origin?

67. Two particles are traveling through space. At time t the first particle is at the point $(-1 + t, 4 - t, -1 + 2t)$ and the second particle is at $(-7 + 2t, -6 + 2t, -1 + t)$.

(a) Describe the two paths in words.
(b) Do the two particles collide? If so, when and where?
(c) Do the paths of the two particles cross? If so, where?

68. For $t > 0$, a particle moves along the curve $x = a + b\sin kt$, $y = a + b\cos kt$, where a, b, k are positive constants.

(a) Describe the motion in words.
(b) What is the effect on the curve of the following changes?
(i) Increasing b
(ii) Increasing a
(iii) Increasing k
(iv) Setting a and b equal