Exercises and Problems for Section 8.3

Exercises

Convert the polar coordinates in Exercises 1–4 to Cartesian coordinates. Give exact answers.

1. $(1, 2\pi/3)$

2. $(\sqrt{3}, -3\pi/4)$

3. $(2\sqrt{3}, -\pi/6)$

4. $(2, 5\pi/6)$

Convert the Cartesian coordinates in Exercises 5–8 to polar coordinates.

5. $(1, 1)$

6. $(-1, 0)$

7. $(\sqrt{6}, -\sqrt{2})$

8. $(-\sqrt{3}, 1)$

9. **(a)** Make a table of values for the equation $r = 1 - \sin\theta$. Include $\theta = 0, \pi/3, \pi/2, 2\pi/3, \pi, \cdots$.
(b) Use the table to graph the equation $r = 1 - \sin\theta$ in the xy-plane. This curve is called a *cardioid*.
(c) At what point(s) does the cardioid $r = 1 - \sin\theta$ intersect a circle of radius 1/2 centered at the origin?
(d) Graph the curve $r = 1 - \sin 2\theta$ in the xy-plane. Compare this graph to the cardioid $r = 1 - \sin\theta$.

10. Graph the equation $r = 1 - \sin(n\theta)$, for $n = 1, 2, 3, 4$. What is the relationship between the value of n and the shape of the graph?

11. Graph the equation $r = 1 - \sin\theta$, with $0 \le \theta \le n\pi$, for $n = 2, 3, 4$. What is the relationship between the value of n and the shape of the graph?

12. Graph the equation $r = 1 - n\sin\theta$, for $n = 2, 3, 4$. What is the relationship between the value of n and the shape of the graph?

13. Graph the equation $r = 1 - \cos\theta$. Describe its relationship to $r = 1 - \sin\theta$.

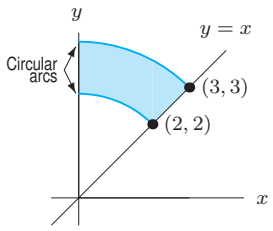
14. Give inequalities that describe the flat surface of a washer that is one inch in diameter and has an inner hole with a diameter of 3/8 inch.

15. Graph the equation $r = 1 - \sin(2\theta)$ for $0 \le \theta \le 2\pi$. There are two loops. For each loop, give a restriction on θ that shows all of that loop and none of the other loop.

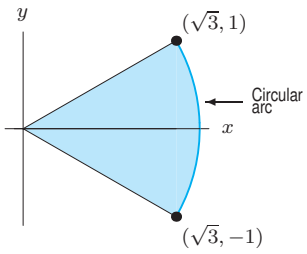
16. A slice of pizza is one eighth of a circle of radius 1 foot. The slice is in the first quadrant, with one edge along the x-axis, and the center of the pizza at the origin. Give inequalities describing this region using:
(a) Polar coordinates **(b)** Rectangular coordinates

In Exercises 17–19, give inequalities for r and θ which describe the following regions in polar coordinates.

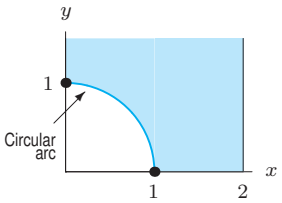
17.



18.



19.



Note: Region extends indefinitely in the y-direction.

20. Find the slope of the curve $r = 2$ at $\theta = \pi/4$.

21. Find the slope of the curve $r = e^{\theta}$ at $\theta = \pi/2$.

22. Find the slope of the curve $r = 1 - \cos\theta$ at $\theta = \pi/2$.

23. Find the arc length of the curve $r = e^{\theta}$ from $\theta = \pi/2$ to $\theta = \pi$.

24. Find the arc length of the curve $r = \theta^2$ from $\theta = 0$ to $\theta = 2\pi$.

Problems

25. Sketch the polar region described by the following integral expression for area:

$$\frac{1}{2}\int_0^{\pi/3} \sin^2(3\theta)\, d\theta.$$

26. Find the area inside the spiral $r = \theta$ for $0 \le \theta \le 2\pi$.

27. Find the area between the two spirals $r = \theta$ and $r = 2\theta$ for $0 \le \theta \le 2\pi$.

28. Find the area inside the cardioid $r = 1 + \cos\theta$ for $0 \le \theta \le 2\pi$.