Each function in Exercises 4–7 has a critical point at $(0, 0)$. What sort of critical point is it?

4. $g(x, y) = x^4 + y^3$

5. $f(x, y) = x^6 + y^6$

6. $k(x, y) = \sin x \sin y$

7. $h(x, y) = \cos x \cos y$

For Exercises 8–20, find the critical points and classify them as local maxima, local minima, saddle points, or none of these.

8. $f(x, y) = x^2 - 2xy + 3y^2 - 8y$

9. $f(x, y) = 5 + 6x - x^2 + xy - y^2$

10. $f(x, y) = x^2 - y^2 + 4x + 2y$

11. $f(x, y) = 400 - 3x^2 - 4x + 2xy - 5y^2 + 48y$

12. $f(x, y) = 15 - x^2 + 2y^2 + 6x - 8y$

13. $f(x, y) = x^2y + 2y^2 - 2xy + 6$

14. $f(x, y) = 2x^3 - 3x^2y + 6x^2 - 6y^2$

15. $f(x, y) = x^3 - 3x + y^3 - 3y$

16. $f(x, y) = x^3 + y^3 - 3x^2 - 3y + 10$

17. $f(x, y) = x^3 + y^3 - 6y^2 - 3x + 9$

18. $f(x, y) = (x + y)(xy + 1)$

19. $f(x, y) = 8xy - \frac{1}{4}(x + y)^4$

20. $f(x, y) = e^{2x^2+y^2}$

Problems

21. Find A and B so that $f(x, y) = x^2 + Ax + y^2 + B$ has a local minimum value of 20 at $(1, 0)$.

22. For $f(x, y) = x^2 + xy + y^2 + ax + by + c$, find values of a, b, and c giving a local minimum at $(2, 5)$ and so that $f(2, 5) = 11$.

23. **(a)** Find critical points for $f(x, y) = e^{-(x-a)^2-(y-b)^2}$.
(b) Find a and b such that the critical point is at $(-1, 5)$.
(c) For the values of a and b in part (b), is $(-1, 5)$ a local maximum, local minimum, or a saddle point?

24. Let $f(x, y) = kx^2 + y^2 - 4xy$. Determine the values of k (if any) for which the critical point at $(0, 0)$ is:

(a) A saddle point
(b) A local maximum
(c) A local minimum

For Problems 25–27, use the contours of f in Figure 15.18.

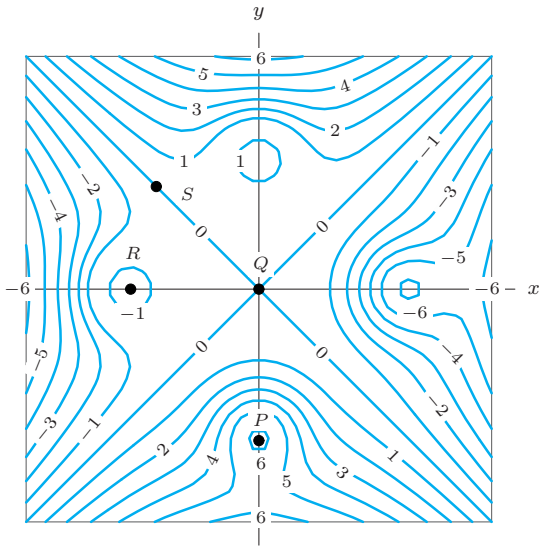

Figure 15.18

25. Decide whether you think each point is a local maximum, local minimum, saddle point, or none of these.

(a) P **(b)** Q **(c)** R **(d)** S

26. Sketch the direction of ∇f at several points around each of P, Q, and R.

27. At the points where $\|\nabla f\|$ is largest, put arrows showing the direction of ∇f.

For Problems 28–31, find critical points and classify them as local maxima, local minima, saddle points, or none of these.

28. $f(x, y) = x^3 + e^{-y^2}$

29. $f(x, y) = \sin x \sin y$

30. $f(x, y) = 1 - \cos x + y^2/2$

31. $f(x, y) = e^x(1 - \cos y)$

32. At the point $(1, 3)$, suppose that $f_x = f_y = 0$ and $f_{xx} > 0$, $f_{yy} > 0$, $f_{xy} = 0$.

(a) What can you conclude about the behavior of the function near the point $(1, 3)$?
(b) Sketch a possible contour diagram.

33. At the point (a, b), suppose that $f_x = f_y = 0$, $f_{xx} > 0$, $f_{yy} = 0$, $f_{xy} > 0$.

(a) What can you conclude about the shape of the graph of f near the point (a, b)?
(b) Sketch a possible contour diagram.

34. Draw a possible contour diagram of f such that $f_x(-1, 0) = 0$, $f_y(-1, 0) < 0$, $f_x(3, 3) > 0$, $f_y(3, 3) > 0$, and f has a local maximum at $(3, -3)$.

35. Draw a possible contour diagram of a function with a saddle point at $(2, 1)$, a local minimum at $(2, 4)$, and no other critical points. Label the contours.